Low Complexity Schemes for the Random Access Gaussian Channel

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Massive Connectivity

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Providing multiple-access to massive number of infrequently UNCOORDINATED communicating devices

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- Very large number of users, say $K_{\text{tot}} \approx 10^6 - 10^7$
- Much smaller, but still large, number of them active within each communication block, say $K_a \approx 50 - 500$
- Message of each active user is short, say $k = 100$ bits
- Blocklength is $n \approx 10^4 - 10^5$
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The goal is to communicate with the smallest possible energy-per-bit

Gaussian Random Access Channel

$$\mathbf{y} = \sum_{i=1}^{K_{\text{tot}}} s_i \mathbf{x}_i + \mathbf{z},$$

- K_{tot} possible users, K_a of them are active
- $(s_1, \ldots, s_{K_{\text{tot}}}) \in \{0,1\}^{K_{\text{tot}}}$ with Hamming weight K_a, unknown
- $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$, $\mathbf{x}_i \in \mathbb{R}^n$, $\|\mathbf{x}_i\|^2 \leq nP$
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Formulation does not become trivial for $K_{tot} = \infty$

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Goal: For given (n, k, K_a, P_e) minimize $\frac{E_b}{N_0} \triangleq \frac{nP}{2k}$

Potential Coding Schemes

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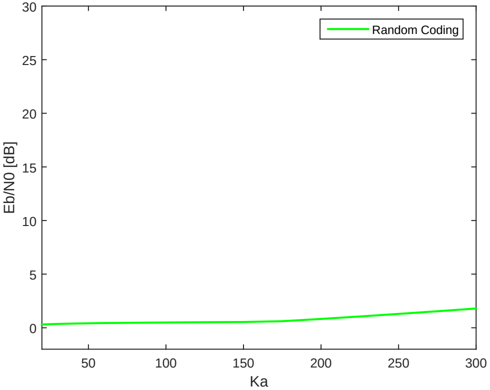
Random coding achievability bound (Polyanskiy, ISIT'17)



$n = 30,000,\ k = 100,\ P_e = 0.05$

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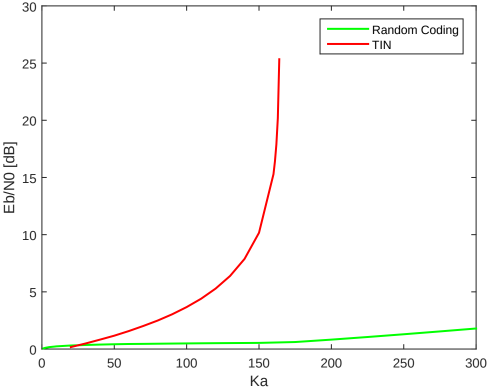
Option I - treat interference as noise (un-coordinated CDMA)

Total spectral efficiency is limited $\rho \triangleq \frac{K_a \cdot k}{n} < \frac{\log(e)}{2} \frac{\text{bits}}{\text{channel use}}$



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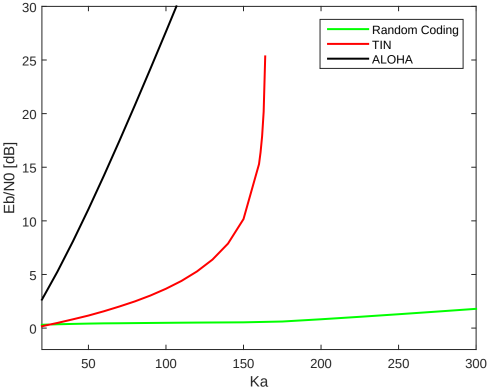
What about more practical solutions?

Option II - slotted-ALOHA

Works well for $P_e \geq 1 - \frac{1}{e}$. Otherwise, to keep collision probability below P_e the fraction of utilized slots is $\approx \ln \frac{1}{1-P_e}$



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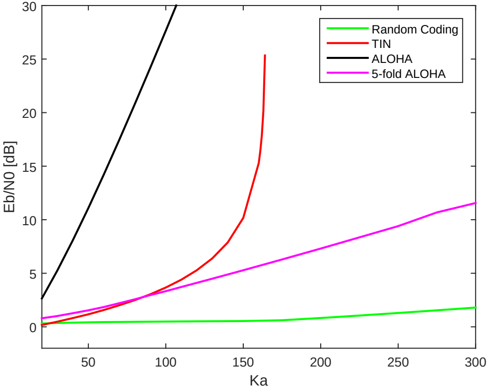
T-fold ALOHA : ALOHA with good channel codes for T-user MAC

Collisions of $\leq T$ users can be resolved

$\Longrightarrow$ fraction of utilized slots increases



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Potential Coding Schemes

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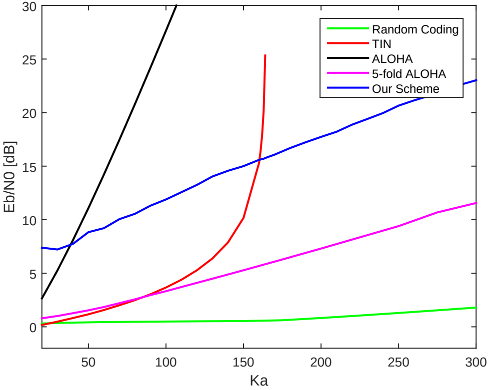
Low-complexity (suboptimal) code design for the T-user MAC with same codebook for all users, combined with T-fold ALOHA



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Split the n channel uses to V sub-blocks

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Coding task: design a codebook $\mathcal{C}$ with efficient encoding/decoding algorithms and good performance for T-user MAC

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We will use a **concatenated code**:

Inner code will deal with noise *(CoF phase)*

Outer code with interaction between codewords *(BAC phase)*

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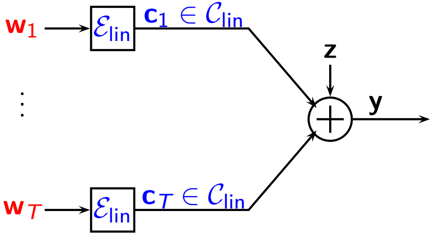

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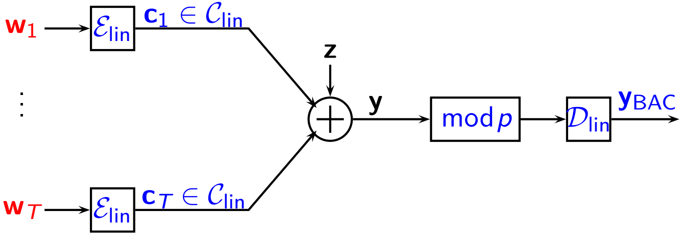

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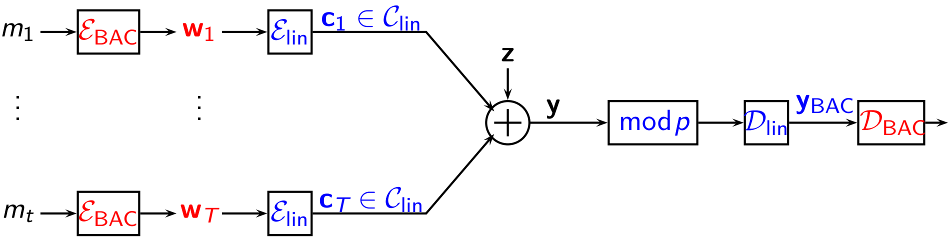
Outer code $\mathcal{C}_{\text{BAC}}$ is designed for the T-user modulo-p adder MAC



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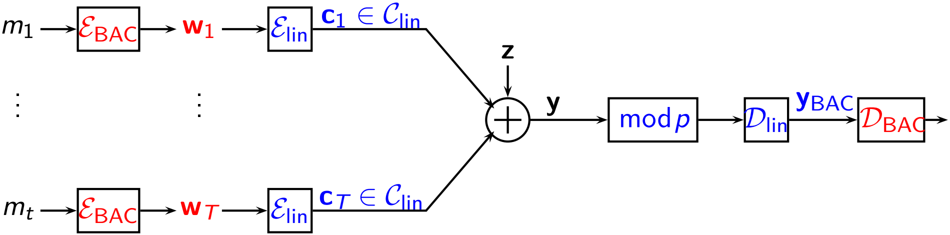
The most practical choice is $p = 2$ and this is our focus



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Related Work

Most of the ideas we use appeared before in various contexts. Their combination to a coding scheme for the Gaussian random access channel is new.

- Compute-and-forward [Nazer-Gastpar'11]
- Explicit codes for the modulo-2 binary adder channel [Lindström'69, Bar-David et al.'93]
- Concatenation of codes with good minimum distance and codes for the BAC [Ericson-Levenshtein'94]
- Concatenation of CoF inner codes with syndrome decoding for compressed sensing [Lee-Hong'16]

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$$\mathbf{y}_{\text{CoF}} = [\mathbf{x} + \mathbf{z}] \bmod 2$$

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***T*-fold ALOHA reduces "power-loss" to $1/T$ instead of $1/K_a$**

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The symmetric-capacity of this MAC is $1/T$ bits/channel use

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How to construct explicit codes?

- Let $H = [\mathbf{h}_1 | \cdots | \mathbf{h}_n]$ be the parity-check matrix of a $[n, k]$ binary T-error correcting code
- All linear combinations of at most T columns are distinct
- The code $\mathcal{C}_{\text{BAC}} = \{\mathbf{h}_1, \ldots, \mathbf{h}_n\}$ is (almost) zero-error for the T-user mod-2 adder MAC
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$\forall r \geq 3,\ \exists [n = 2^r - 1, n - k = rT, d \geq 2T + 1]$ binary BCH code
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Problem: decoding complexity of BCH linear in $n = 2^r - 1$

More on the BAC Phase - Encoding & Decoding

Encoding:

Each column of H is of the form $[\alpha, \alpha^3, \ldots, \alpha^{2T-1}]$, for $\alpha \in GF(2^k) \setminus \{0\}$

To encode, just map messages to elements of $GF(2^k) \setminus \{0\}$ and compute first T odd powers

More on the BAC Phase - Encoding & Decoding

Decoding:

Modified GPZ algorithm, almost the same as Bar-David et al.'93

- *Syndrome computation:* odd syndromes given "for free" from $\mathbf{y}_{\mathrm{BAC}}$. Computing even syndromes from them is easy
- *Construction of error locator polynomial:* Berlekamp-Massey algorithm gives

$$\sigma(X) = 1 + \sum_{t=1}^{L} \sigma_t X^t = \prod_{i=1}^{T} (1 + \alpha_i X)$$

  where $\alpha_1, \ldots, \alpha_T$ correspond to the messages
- *Finding the roots of $\sigma(X)$:* Rabin's probabilistic algorithm [Rabin'80] finds $\alpha_1^{-1}, \ldots, \alpha_T^{-1}$
- *Inversion of the roots:* using the identity $\alpha^{-1} = \alpha^{2^k} - 1$

Total complexity: $\mathcal{O}(kT^2 \log^2(T) \log\log(T))$ operations in $GF(2^k)$

Spectral Efficiency > 1

The spectral efficiency $\rho = \frac{K_a \cdot k}{n}$ of our scheme is at most R_{lin}
What if $\rho > 1$?

Option I - work with $p > 2$

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- BAC phase can be implemented using $H = [\mathbf{h}_1 | \cdots | \mathbf{h}_n]$ of a $[n = p^s - 1, n - k = 2T]$ Reed-Solomon code over $\mathbb{F}_{p^s}$ with

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But encoders in our setup must be extremely simple
$\Longrightarrow$ binary codes are preferable

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Option II - use a multilevel code based on binary codes

- Allows to increase R_{lin} above 1
- Requires some overhead in order to "pair" messages from different layers

Approximate performance

A TDMA scheme with infinite blocklength and fixed K_a can achieve $\left(\frac{E_b}{N_0}\right)^* = \frac{2^{2\rho}-1}{2\rho}$ where $\rho = \frac{K_a \cdot k}{n}$.

How far are we from $\left(\frac{E_b}{N_0}\right)^*$?

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Linear code required to have rate $R_{\text{lin}} = \frac{\rho}{\alpha}$

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How far are we from $\left(\frac{E_b}{N_0}\right)^*$?

Fix T and set the number of sub-blocks to $V = \frac{K_a}{\alpha T}$ for $\alpha \in (0,1]$.

The T-collision probability is $\Pr\left(\mathrm{Binomial}\left(K_a - 1, \frac{\alpha T}{K_a}\right) \geq T\right)$

Linear code required to have rate $R_{\text{lin}} = \frac{\rho}{\alpha}$

$$\Delta = \left(\frac{E_b}{N_0}\right) \mathrm{dB} - \left(\frac{E_b}{N_0}\right)^* \mathrm{dB}$$

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Linear code required to have rate $R_{\mathrm{lin}} = \frac{\rho}{\alpha}$

$$\Delta = \left(\frac{E_b}{N_0}\right) \mathrm{dB} - \left(\frac{E_b}{N_0}\right)^* \mathrm{dB}$$
$$\approx 6\rho\frac{1-\alpha}{\alpha} + 10\log_{10}(\alpha)$$

T-Collision avoidance loss due to a $1/\alpha$ increase in spectral efficiency

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$$\approx 6\rho\frac{1-\alpha}{\alpha} + 10\log_{10}(\alpha) + 10\log_{10}(T)$$

CoF loss from the reduction $\mathbf{y} \mapsto \mathbf{y}_{\mathsf{CoF}}$

Approximate performance

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$$\approx 6\rho\frac{1-\alpha}{\alpha} + 10\log_{10}(\alpha) + 10\log_{10}(T) - 10\log_{10}(1-2^{-2\rho})$$

Loss of $+1$ in computation rate

Approximate performance

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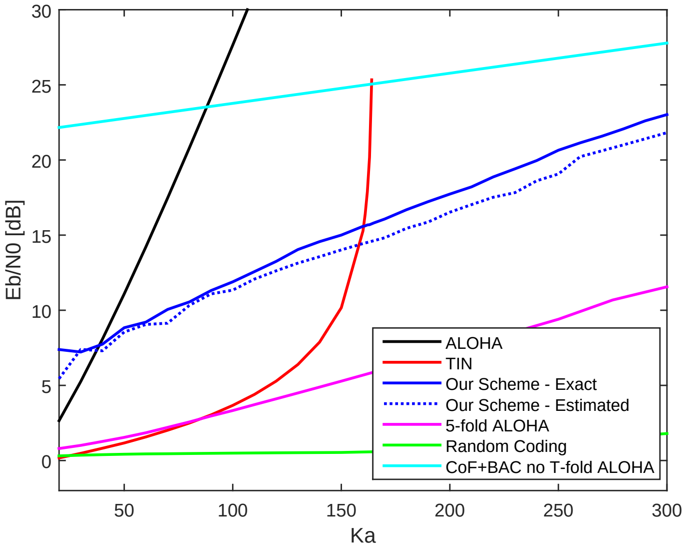
Linear code required to have rate $R_{\text{lin}} = \frac{\rho}{\alpha}$

$$\Delta = \left(\frac{E_b}{N_0}\right)\mathrm{dB} - \left(\frac{E_b}{N_0}\right)^*\mathrm{dB}$$
$$\approx 6\rho\frac{1-\alpha}{\alpha} + 10\log_{10}(\alpha) + 10\log_{10}(T) - 10\log_{10}(1-2^{-2\rho}) + 1.53$$

Shaping loss

Approximate performance



Summary and Conclusions

- We considered T-fold ALOHA (MPR) as a candidate coding scheme for the Gaussian random access channel
- We constructed a practical variant of T-fold ALOHA based on concatenation of CoF codes and BAC codes
- Our scheme is far from optimal, but significantly outperforms competing low-complexity schemes in regimes of practical interest