Ozarow-Type Outer Bounds for Memoryless Sources and Channels

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Joint work with Yuval Kochman (HUJI) and Yury Polyanskiy (MIT)

ISIT,
Vail, Colorado
June 21, 2018

This talk

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A unified framework for obtaining (relatively simple) outer bounds for the multiple description and the JSCC broadcast problems

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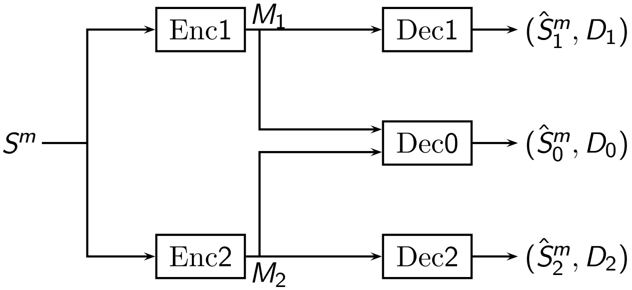
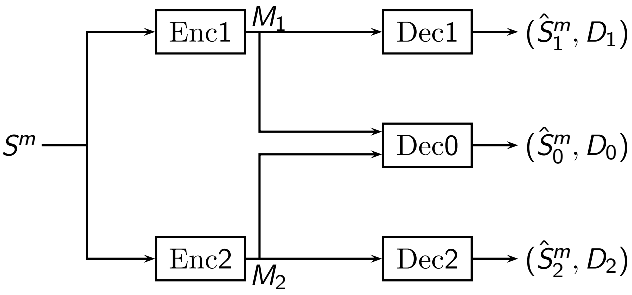
What will we **not** have in this talk?

Improved bounds

Multiple Description Problem

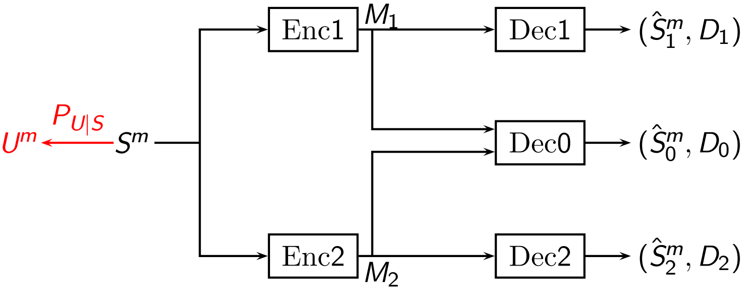


Multiple Description Problem



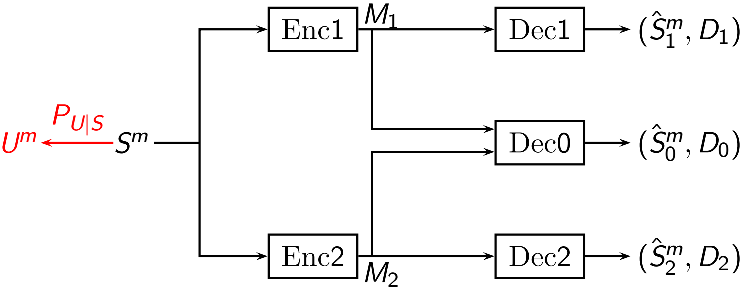
Nontrivial tradeoff between $(R_1, R_2, D_0, D_1, D_2)$
How to capture this?

Multiple Description Problem



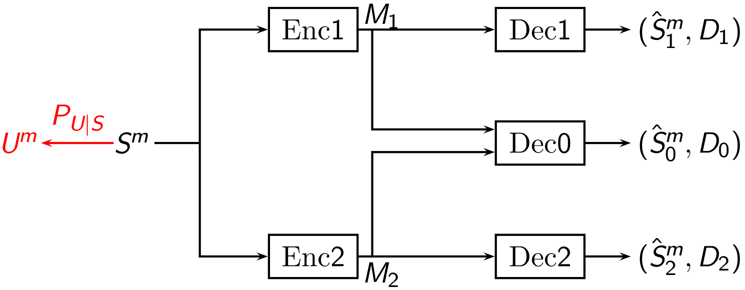
Ozarow's trick for converse:
Bound $R_1 + R_2$ w.r.t. an auxiliary U^m generated by the DMC $P_{U|S}$

Multiple Description Problem



$$
\begin{aligned}
m(R_1 + R_2) \geq\ & I(S^m; M_1, M_2) - I(U^m; M_1, M_2) \\
& + I(U^m; M_1) + I(U^m; M_2) + I(M_1; M_2 | U^m)
\end{aligned}
$$

Multiple Description Problem



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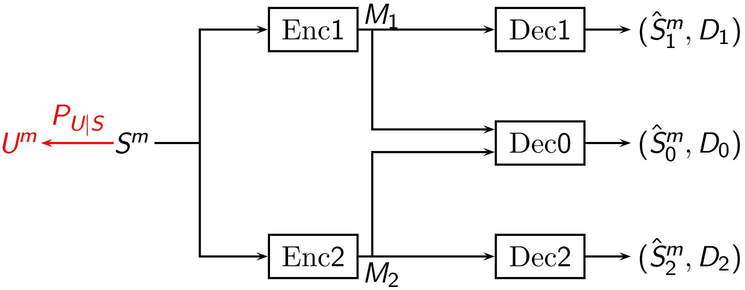
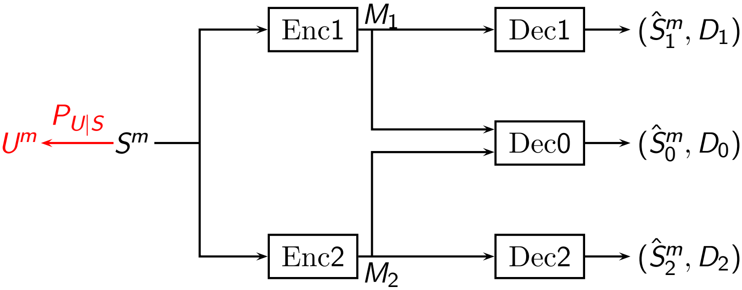
Multiple Description Problem



$$m(R_1 + R_2) \geq m(h(S) - h(U)) + [h(U^m|M_1, M_2) - h(S^m|M_1, M_2)] \\ + I(U^m; M_1) + I(U^m; M_2)$$

For S Gaussian, $U = S + N$, N Gaussian, $D = \mathrm{MSE}$

Multiple Description Problem



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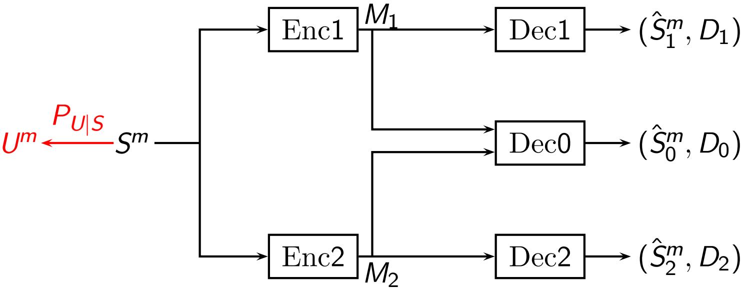
For S Gaussian, $U = S + N$, N Gaussian, $D = \mathrm{MSE}$

Bound $h(U^m|M_1, M_2) - h(S^m|M_1, M_2)$ using EPI

Multiple Description Problem

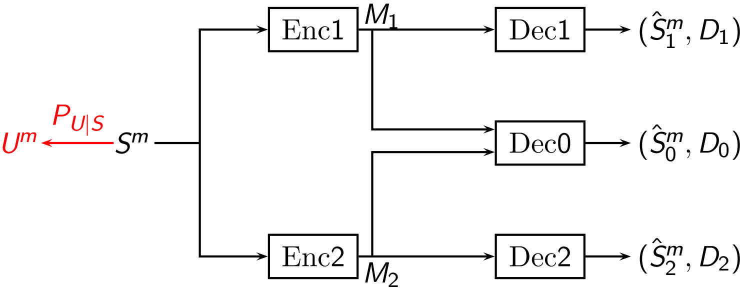


$$m(R_1 + R_2) \geq m(h(S) - h(U)) + [h(U^m|M_1, M_2) - h(S^m|M_1, M_2)] + I(U^m; M_1) + I(U^m; M_2)$$

For S Gaussian, $U = S + N$, N Gaussian, $D = \mathrm{MSE}$

Bound $I(U^m; M_i) \geq I(U^m; \hat{S}_i^m)$ using SLB

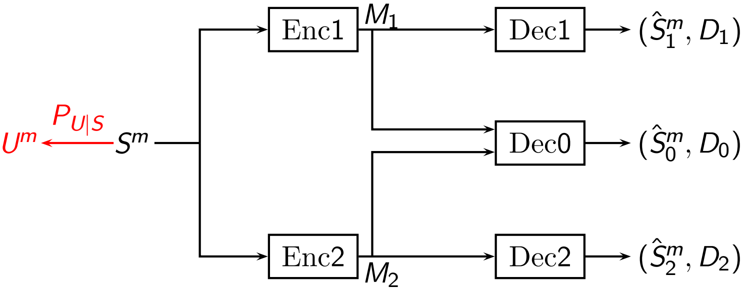
Multiple Description Problem



$$m(R_1 + R_2) \geq I(S^m; M_1, M_2) - I(U^m; M_1, M_2) + I(U^m; M_1) + I(U^m; M_2)$$

Let's go one step backwards

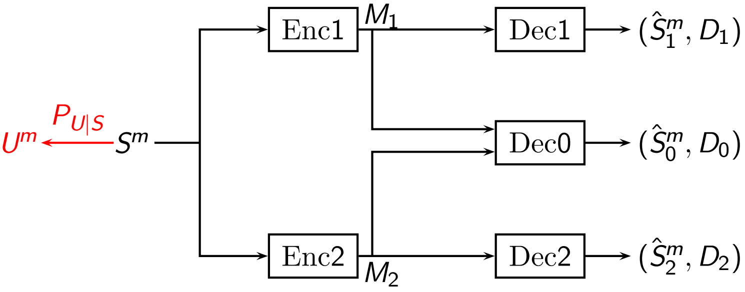
Multiple Description Problem



$$\begin{aligned} m(R_1 + R_2) \geq\ & I(S^m; M_1, M_2 | U) \\ & + I(U^m; M_1) + I(U^m; M_2) \end{aligned}$$

By Markovity $U^m - S^m - (M_1, M_2)$

Multiple Description Problem



$$m(R_1 + R_2) \geq I(S^m; \hat{S}_0^m | U^m) + I(U^m; \hat{S}_1^m) + I(U^m; \hat{S}_2^m)$$

By DPI

Multiple Description Problem

$$U^m - S^m - \hat{S}_0^m$$

$$m(R_1 + R_2) \geq I(S^m; \hat{S}_0^m | U^m) + I(U^m; \hat{S}_1^m) + I(U^m; \hat{S}_2^m)$$

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Let $P = P_{SU} = P_S P_{U|S}$ and define

$$F_P(t) \triangleq \min_{\substack{V \,:\, U - S - V \\ I(S;V) \geq t}} I(S; V | U)$$

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Lemma

The function $F_P(t)$ is monotone non-decreasing, convex, and tensorizes, i.e., $F_{P^{\otimes m}}(mt) = m \cdot F_P(t)$.

Multiple Description Problem

$$U^m - S^m - \hat{S}_0^m$$

$$m(R_1 + R_2) \geq F_{P^{\otimes m}}(I(S^m; \hat{S}_0^m)) + I(U^m; \hat{S}_1^m) + I(U^m; \hat{S}_2^m)$$

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Let $P = P_{SU} = P_S P_{U|S}$ and define

$$\bar{R}_P(D) \triangleq \min_{\substack{\hat{S} \,:\, U - S - \hat{S} \\ \mathbb{E}d(S,\hat{S}) \leq D}} I(U; \hat{S})$$

Multiple Description Problem

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Let $\hat{S}^m$ be a random vector satisfying the Markov chain $U^m - S^m - \hat{S}^m$ and $\mathbb{E}d(S^m, \hat{S}^m) \leq D$, then $I(U^m; \hat{S}^m) \geq m\bar{R}_P(D)$

Multiple Description Problem

$$U^m - S^m - \hat{S}_0^m$$

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Multiple Description Problem

Theorem

If $(R_1, R_2, D_0, D_1, D_2)$ are achievable for the MD problem, then for any choice of $P_{U|S}$ we have

$$R_1 + R_2 \geq F_P(R(D_0)) + \bar{R}_P(D_1) + \bar{R}_P(D_2)$$

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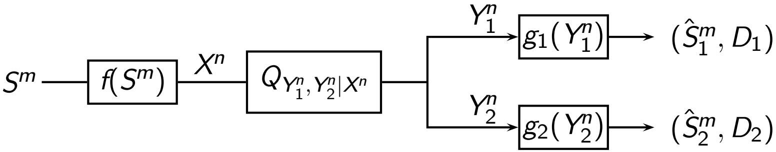
Remarks:

- Agrees with Ozarow for quadratic Gaussian
- Can be obtained as a corollary of [Song-Shao-Chen IT'14]

JSCC Over Broadcast



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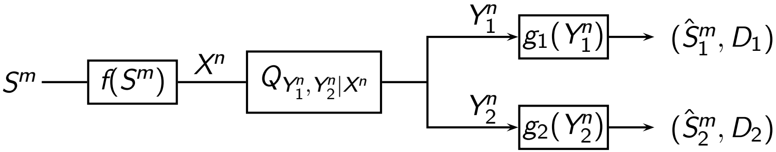

Nontrivial tradeoff between (D_1, D_2)
How to capture this?

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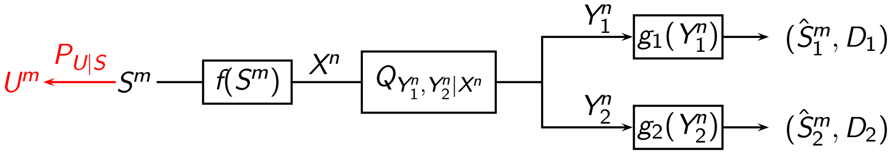

Ozarow/Reznic-Feder-Zamir trick for outer bound:
Bound (D_1, D_2) tradeoff w.r.t. an auxiliary U^m generated by the DMC $P_{U|S}$

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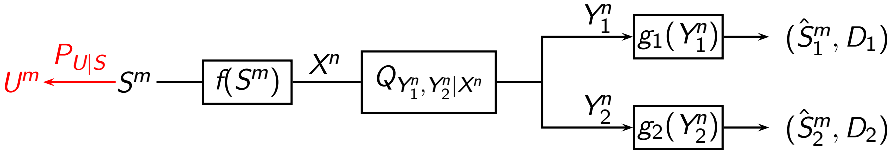

$$m \cdot \bar{R}_P(D_2) \leq I(U^m; \hat{S}_2^m)$$

$$\bar{R}_P(D) \triangleq \min_{\substack{\hat{S} \,:\, U - S - \hat{S} \\ \mathbb{E}d(S, \hat{S}) \leq D}} I(U; \hat{S})$$

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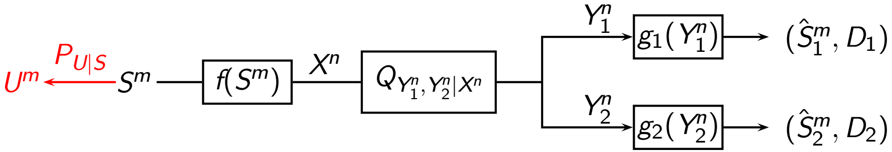
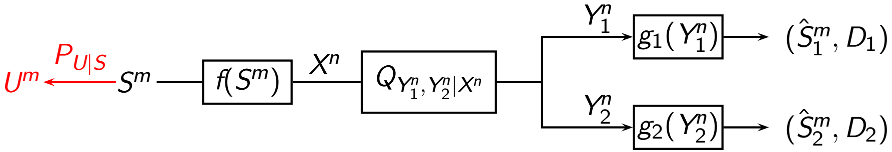


$$m \cdot \bar{R}_P(D_2) \le I(U^m; \hat{S}_2^m)$$
$$\le I(U^m; Y_2^n)$$

DPI

JSCC Over Broadcast



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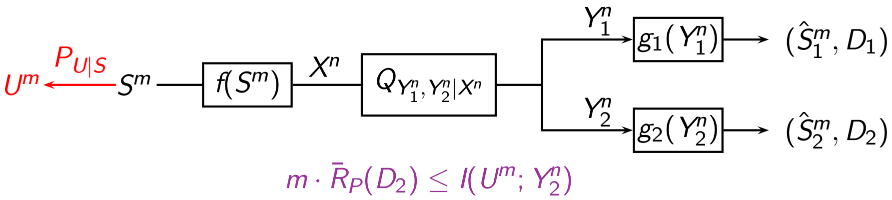
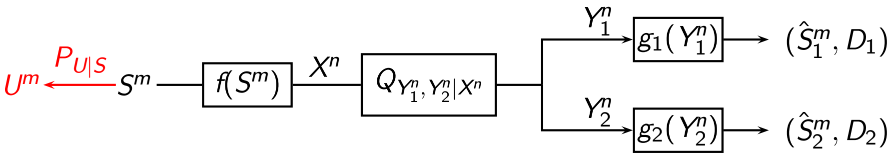


Let $Q^n = Q^n_{Y_1^n, Y_2^n | X^n}$.

$$G_{Q^n}(t) \triangleq \max_{\substack{W, X^n \;:\; W - X^n - (Y_1^n, Y_2^n) \\ I(X^n; Y_1^n | W) \geq t}} I(Y_2^n; W)$$

JSCC Over Broadcast



$$
\begin{aligned}
m \cdot \bar{R}_P(D_2) &\leq I(U^m; Y_2^n) \\
&\leq G_{Q^n}\left(I(X^n; Y_1^n | U^m)\right)
\end{aligned}
$$

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$$
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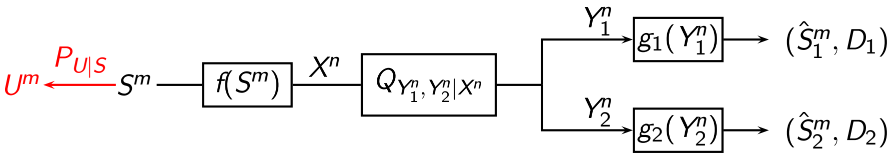
$$

Note that in our case $W = U^m$ which can be highly suboptimal

JSCC Over Broadcast



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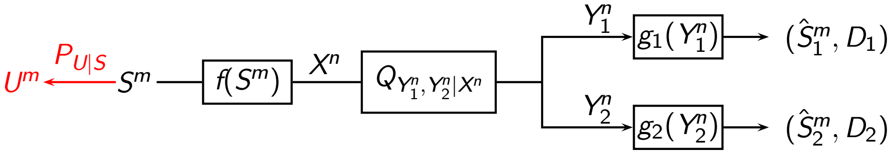
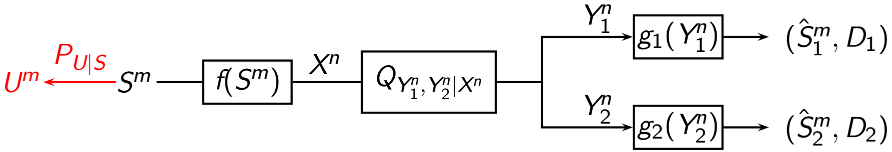


$$\begin{aligned} m \cdot \bar{R}_P(D_2) &\leq G_{Q^n}\left(I(X^n; Y_1^n | U^m)\right) \\ &\leq G_{Q^n}\left(I(S^m; Y_1^n | U^m)\right) \end{aligned}$$

DPI

JSCC Over Broadcast



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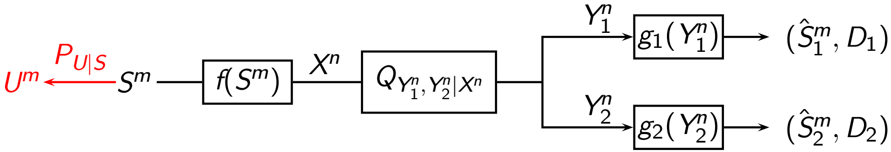

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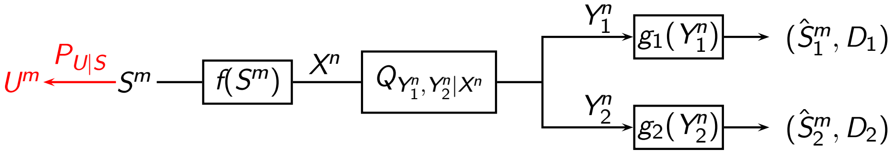
JSCC Over Broadcast



$$\begin{aligned} m \cdot \bar{R}_P(D_2) &\leq G_{Q^n}\left(I(S^m; Y_1^n | U^m)\right) \\ &\leq G_{Q^n}\left(F_{P^{\otimes m}}\left(I(S^m; Y_1^n)\right)\right) \end{aligned}$$

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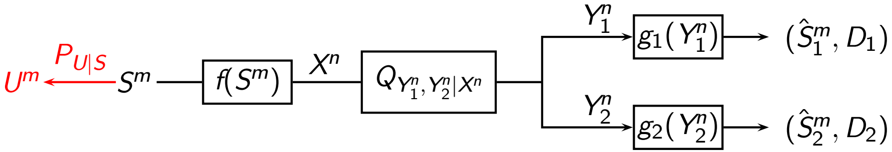

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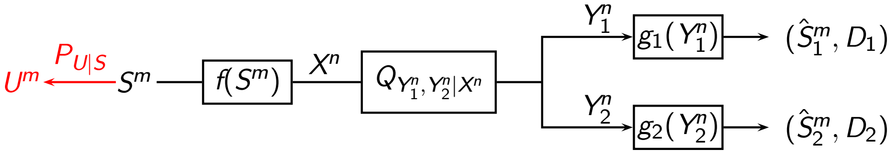

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Lemma

The function $F_P(t)$ is monotone non-decreasing, convex, and tensorizes, i.e., $F_{P^{\otimes m}}(mt) = m \cdot F_P(t)$.

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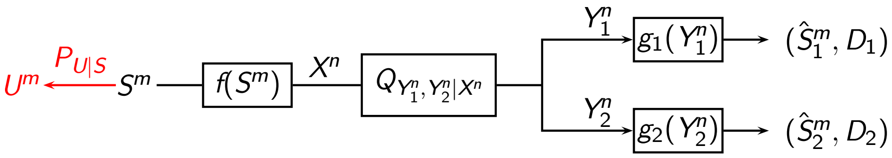

$$
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m \cdot \bar{R}_P(D_2) &\le G_{Q^n}\left(F_{P^{\otimes m}}\left(I(S^m; Y_1^n)\right)\right) \\
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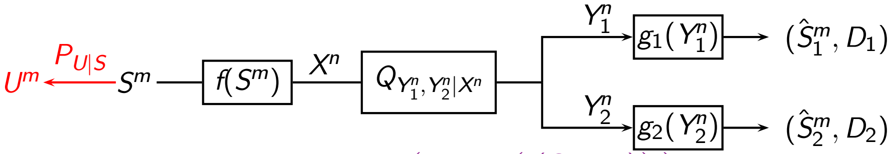

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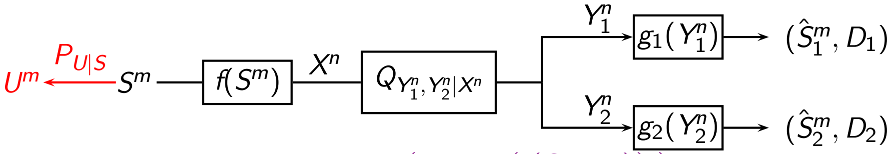

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m \cdot \bar{R}_P(D_2) &\leq G_{Q^n}\left(m \cdot F_P \left(\frac{I(S^m; Y_1^n)}{m} \right) \right) \\
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$$

DPI

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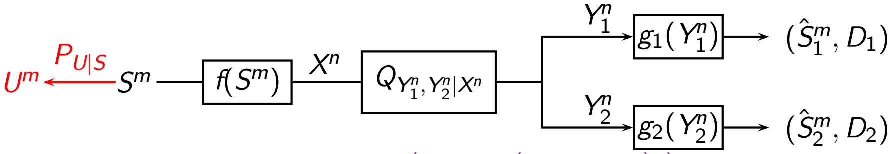

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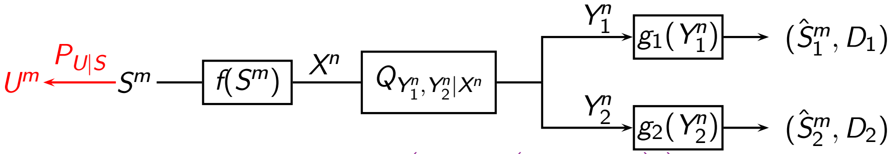
JSCC Over Broadcast



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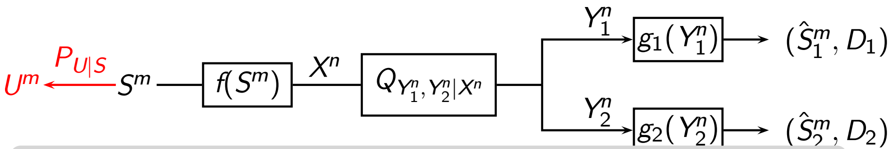
RDF

JSCC Over Broadcast



Theorem

If (D_1, D_2) are achievable over the broadcast Q^n, then for any choice of $P_{U|S}$ we have

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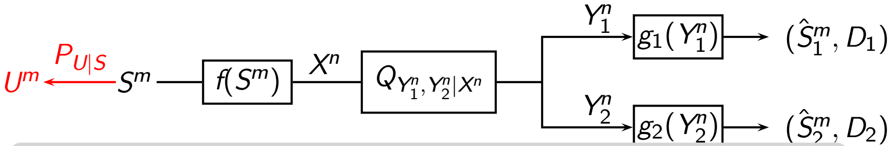
JSCC Over Broadcast



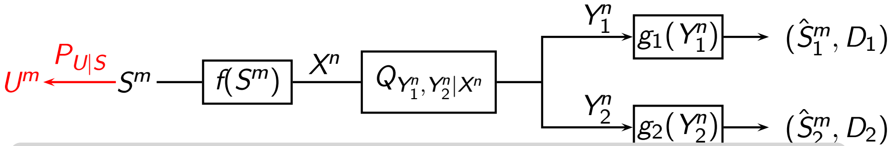
Theorem

If (D_1, D_2) are achievable over the degraded memoryless broadcast $Q^{\otimes n}$, then for any choice of $P_{U|S}$ we have $\rho = n/m$

$$\bar{R}_P(D_2) \leq \rho G_Q \left(\frac{F_P\left(R(D_1)\right)}{\rho} \right)$$

If Q^n is degraded memoryless BC, then $G_{Q^{\otimes n}}(nt) = nG_Q(t)$

JSCC Over Broadcast



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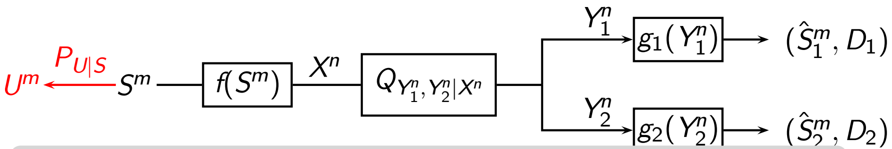
In this case:

- The BC capacity region boundary is $(C_1, G_Q(C_1))$
- The maximizing W in definition of $G_{Q^{\otimes n}}$ is n-letter iid, so bound can't be tight for $m \neq n$

JSCC Over Broadcast



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- For Quadratic Gaussian over GBC, recovers [Reznic-Feder-Zamir, IT'06]
- Can be obtained as a corollary of [Khezeli-Chen IT'15]

Example

- $S \sim \mathrm{Bernoulli}(1/2)$ i.i.d
- Hamming distortion
- $Y_1 = X \oplus Z_1,\ Y_2 = Y_1 \oplus Z_2$
- $Z_1 \sim \mathrm{Bernoulli}(\delta_1),\ Z_2 \sim \mathrm{Bernoulli}(\delta_2)$

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The choice $U = S \oplus N$, $N \sim \mathrm{Bernoulli}(q)$ gives:

Theorem

for any $0 \leq q \leq 1/2$, it holds that

$$\begin{aligned} &\log 2 - h_b(q * D_2) \\ &\leq \rho \left[\log 2 - h_b \left(\delta_2 * h_b^{-1} \left(h_b(\delta_1) + \frac{h_b(q * D_1) - h_b(D_1)}{\rho} \right) \right) \right] \end{aligned}$$

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In particular, for $D_2 = D_2^* = h_b^{-1}(\log 2 - \rho(\log 2 - h_b(\delta_1 * \delta_2)))$, taking $q \to 0$ yields

Example

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Theorem

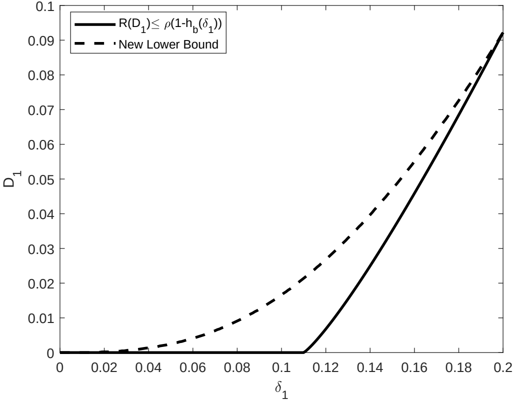
If (D_1, D_2^*) is achievable, then

$$g(D_1) \geq \frac{g(\delta_1)}{g(\delta_1 * \delta_2)} \cdot g(D_2^*),$$

where $g(t) \triangleq (1-2t)\log\left(\frac{1-t}{t}\right)$.

Example

- $S \sim \mathrm{Bernoulli}(1/2)$ i.i.d, $\rho = 2$
- Hamming distortion
- $Y_1 = X \oplus Z_1,\ Y_2 = Y_1 \oplus Z_2$
- $Z_1 \sim \mathrm{Bernoulli}(\delta_1),\ Z_2 \sim \mathrm{Bernoulli}(\delta_2),\ \delta_1 * \delta_2 = 0.2$



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For $D_1 = D_1^* = h_b^{-1}(\log 2 - \rho(\log 2 - h_b(\delta_1)))$, taking $q \to 1/2$ yields

Theorem

If (D_1^*, D_2) is achievable, then

$$D_2 \geq \delta_2 * D_1^*$$

Example

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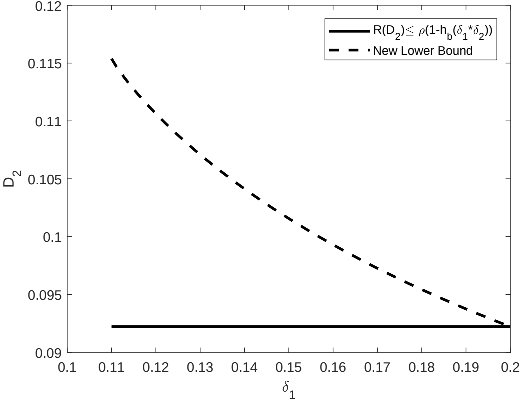

Conclusions

For a source P_S, an auxiliary channel $P_{U|S}$, and a distortion measure $d(\cdot,\cdot)$, we defined two functions:

$$F_P(t) \triangleq \min_{\substack{V\,:\, U - S - V \\ I(S;V) \geq t}} I(S;V|U)$$

$$\bar{R}_P(D) \triangleq \min_{\substack{\hat{S}\,:\, U - S - \hat{S} \\ \mathbb{E}d(S,\hat{S}) \leq D}} I(U;\hat{S})$$

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For memoryless source, the two functions tensorize, and an Ozarow-type single letter outer bound for MD problem was derived, in terms of $F_P(t)$ and $\bar{R}_P(D)$

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For a broadcast channel $Q_{Y_1^n, Y_2^n | X^n}$ we further defined

$$G_{Q^n}(t) \triangleq \max_{\substack{W, X^n \,:\, W - X^n - (Y_1^n, Y_2^n) \\ I(X^n; Y_1^n | W) \geq t}} I(Y_2^n; W)$$

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For degraded memoryless BC $G(\cdot)$ tensorizes, and a single-letter Reznik-Feder-Zamir type bound was derived, in terms of $F_P(t), \bar{R}_P(D)$ and $G_Q(t)$